$\mathscr{L}_1$ Adaptive Control in an Iterative Learning Control Framework: Stability, Robustness and Design Trade-Offs*

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Abstract— **This paper introduces a modified robust iterative control framework with $\mathscr{L}_1$ adaptive feedback for single-input single-output (SISO) linear time invariant (LTI) systems with iteration varying constant parametric uncertainties. The adaptive loop compensates for nonrepetitive effects (exogenous disturbances and/or uncertainties) and ensures that the plant, as seen from the ILC input, is sufficiently close to its nominal value for performance improvement through learning. The $\mathscr{L}_1$ controller is reformulated to accommodate the feedforward input, resulting in an adaptation that considers changes in system response due to learning. A rigorous stability analysis is presented. Performance and trade-offs are evaluated via simulation.**

I. INTRODUCTION

Iterative learning control (ILC) is a feedforward control strategy aimed towards systems that execute the same task repetitively [1]. ILC is based on the idea that the performance of such systems can be improved by using information from previous trials. As opposed to other learning control strategies (e.g. adaptive control, neural networks, repetitive control), ILC modifies the control input rather than the controller itself [1], [2]. As such, ILC can be thought of as feedback control in the iteration domain. Naturally, this property equips iterative learning controllers with simplicity, robustness and fast convergence to iteration domain equilibria with significant decrease in tracking error metrics up to several orders of magnitude. Research on robust iterative learning control has focused on disturbance rejection, stochastic effects, transient growth, μ synthesis, $\mathscr{H}_\infty$ framework, robustness to high frequency modeling uncertainties and design of ILC algorithms for systems with large parametric uncertainties (see [1], [3]–[10]). Robustness of control algorithms in the time and iteration domains is especially important as applications with parametric uncertainties (multi-agent systems, pick and place industrial robotics, prosthetics) requiring monotonic behavior and high tracking performance can benefit from it.

In this paper, we provide a rigorous stability analysis and performance evaluation of the $\mathscr{L}_1$-ILC scheme in [11] on a class of single-input single-output (SISO) linear time invariant (LTI) systems. The $\mathscr{L}_1$ feedback loop is used to compensate for iteration varying constant parametric uncertainties and other nonrepeating effects in the time domain, whereas the ILC improves tracking performance in the iteration domain. That is, the presence of an adaptive feedback controller makes certain the plant, as seen from the feedforward ILC input, remains close to its nominal value. The use of $\mathscr{L}_1$ adaptive control as opposed to more conventional forms (e.g. model reference adaptive control) can be attributed to guaranteed robustness bounds (stability of the feedback loop is a necessary condition for ILC), along with a priori known steady state and transient performance. We modify the $\mathscr{L}_1$ adaptive control architecture to accommodate parallel ILC signals and prevent the trade-off between time and iteration domains previously found in [11].

The rest of the paper is outlined as follows. Section II presents a modified $\mathscr{L}_1$ controller for a class of uncertain systems. An overview of ILC, aspects pertaining to our problem and the details of the learning controller in question are given in section III. Section IV discusses the trade-offs between time and iteration domain properties for the $\mathscr{L}_1$-ILC scheme. Trade-offs and performance specifications are evaluated via simulation in section V. Conclusions and future work are discussed in Section VI.

II. $\mathscr{L}_1$ ADAPTIVE CONTROL

$\mathscr{L}_1$ adaptive control theory is a recently developed methodology [12] with guaranteed transient performance and robustness in the presence of fast adaptation. The critical feature of $\mathscr{L}_1$ adaptive control theory is the decoupling of estimation and control, realized by the insertion of a bandlimited filter at a particular point in the architecture. In $\mathscr{L}_1$ adaptive control, adaptation rates can be increased arbitrarily; although practical concerns such as hardware speed and noise may limit achievable performance. The performance-robustness trade-off of $\mathscr{L}_1$ systems is defined by the bandwidth of the filter and can be adressed with tools from classical and robust control. Consequently, uniform performance bounds on all system signals can be enforced without resorting to gain scheduling, persistency of excitation or high gain feedback.

$\mathscr{L}_1$ adaptive control algorithms have been developed for a wide variety of classes. We now present the $\mathscr{L}_1$ architecture for SISO LTI systems with unknown constant parameters. To put the $\mathscr{L}_1$-ILC problem into a meaningful format, we augment the original controller [12] with a bounded feedforward signal. This makes sure that the ILC signal does not act as a disturbance to the $\mathscr{L}_1$ controller and overcomes the trade-off previously observed in [11].

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A. Problem Formulation

Consider the class of systems

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(u(t) + \theta x(t)), \quad x(0) = x_0, \\ y(t) &= Cx(t). \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the measured state vector; $u(t) \in \mathbb{R}$ is the control input; $B, C^T \in \mathbb{R}^n$ are known constant vectors; $A \in \mathbb{R}^{n\times n}$ is a known constant matrix, with (A,B) controllable; $\theta^T \in \Theta \triangleq \{v \in \mathbb{R}^n : \|v\|_\infty \leq m\}$ is an unknown constant vector, with $m \in \mathbb{R}$; and $y(t) \in \mathbb{R}$ is the output signal. Without loss of generality, assume A to be Hurwitz. The $\mathscr{L}_1$ adaptive controller ensures transient and steady-state behavior in the input and output channels in relation to the $\mathscr{L}_1$ reference system. The reference system is described by the triple (A,B,C), the strictly proper bounded-input bounded-output (BIBO) stable transfer function $D(s)$ with DC gain 1 and zero state space initialization, and the unknown parameter θ. $D(s)$ is also subject to the $\mathscr{L}_1$ norm stability condition

$$\lambda \triangleq \|G(s)\|_{\mathscr{L}_1} \delta < 1, \tag{1}$$

where $G(s) \triangleq H(s)(1-D(s))$; $H(s) \triangleq (s\mathbb{I}-A)^{-1}B$ and $\delta \triangleq \max_{\theta^T \in \Theta} \|\theta\|_1 = mn$; which guarantees bounded-input bounded-state (BIBS) stability of the reference system. The feedforward augmented closed loop reference system can then be defined as

$$\begin{aligned} \dot{x}_{ref}(t) &= Ax_{ref}(t) + B(u_{ref}(t) + \theta x_{ref}(t)), \\ y_{ref}(t) &= Cx_{ref}(t), \\ U_{ref}(s) &= D(s)(K_g R(s) - \theta X_{ref}(s)) + U_i(s), \end{aligned} \tag{2}$$

with initial condition $x_{ref}(0) = x_0$, where $K_g = 1/(CH(0))$ is a static precompensator; $R(s)$ is the reference signal; and $U_i(s)$ is a bounded input signal in Laplace notation. By changing the original reference model [12] to include $U_i(s)$, we seek to avoid the feedforward acting as a disturbance to the closed loop system.

B. $\mathscr{L}_1$ Adaptive Controller

The $\mathscr{L}_1$ adaptive controller is based on the fast estimation scheme which makes use of a state predictor, the bounded feedforward input $u_i(t)$ and the bandlimited filter $D(s)$.

1) State Predictor: The control law relies on the following state predictor

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B(\hat{\theta}(t)x(t) + u(t)) - K_{sp}\tilde{x}(t), \quad \hat{x}(0) = x_0, \tag{3}$$

where $\hat{x}(t)$ is the state prediction vector; $\hat{\theta}^T(t)$ is the estimate of the unknown constant vector θ^T; $\tilde{x}(t) \triangleq \hat{x}(t) - x(t)$ is the prediction error; and $K_{sp} \in \mathbb{R}^{n\times n}$ can be used to assign faster poles to $(A - K_{sp})$ [13].

2) Adaptation Law: The adaptation law that estimates θ is

$$\dot{\hat{\theta}}^T(t) = \Gamma Proj(\hat{\theta}^T(t), -\tilde{x}^T(t)VBx(t)), \tag{4}$$

with initial condition $\hat{\theta}^T(0) = \hat{\theta}_0^T \in \Theta$, where $Proj(.,.)$ is the smooth projection operator defined in [14]; $\Gamma > 0$ is the adaptation rate; and $V = V^T > 0$ is the solution to the algebraic Lyapunov equation $A^TV + VA = -Z$, with arbitrary $Z = Z^T > 0$. The projection operator ensures that $\hat{\theta}^T(t) \in \Theta$ $\forall t \in [0,\infty)$ by definition. This property is used extensively in the analysis of $\mathscr{L}_1$ schemes.

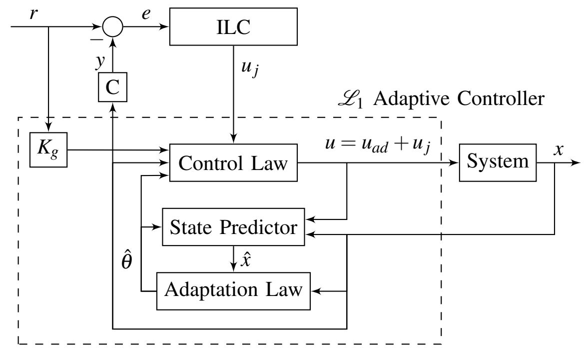


Fig. 1: ILC with modified $\mathscr{L}_1$ adaptive feedback

3) Control Law: The control input is defined as

$$\begin{aligned} u(t) &= u_i(t) + u_{ad}(t), \quad |u_i(t)| \leq M \in \mathbb{R}, \\ U_{ad}(s) &\triangleq D(s)(K_g R(s) - \hat{\eta}(s)), \end{aligned} \tag{5}$$

where $u_i(t)$ and $u_{ad}(t)$ are the feedforward and feedback signals, respectively, and $\hat{\eta}(s)$ is the Laplace transform of $(\hat{\theta}(t)x(t))$. Inclusion of the feedforward signal in the control input leads to the augmentation of the state predictor (see fig. 1). Hence, the controller generates the proper adaptive signal $u_{ad}(t)$ to track (2).

The closed loop system with control (5) defined according to (3) and (4), together with the stability condition (1), is stable. In addition, the system has uniform performance bounds on both the input and the output:

$$\begin{aligned} \|x_{ref} - x\|_{\mathscr{L}_\infty} &\leq \frac{\phi_1}{\sqrt{\Gamma}}, \quad \lim_{t\to\infty}(x_{ref}(t) - x(t)) = 0, \\ \|u_{ref} - u\|_{\mathscr{L}_\infty} &\leq \frac{\phi_2}{\sqrt{\Gamma}}, \quad \lim_{t\to\infty}(u_{ref}(t) - u(t)) = 0, \end{aligned} \tag{6}$$

where ϕ_1 and ϕ_2 are constants dependent on system parameters. In other words, arbitrary close model tracking can be achieved by increasing Γ. As ILC uses information from input and output channels, this property enables the use of the reference model in designing the ILC update law. Moreover, the reference system can be made arbitrarily close to the design system [12] by increasing the bandwidth of $D(s)$. This, however, comes at the expense of reduced robustness.

When a feedforward signal is present in the control, designing the state predictor with $u_{ad}(t)$ instead of $u(t)$ leads to the following prediction error dynamics:

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + B(\tilde{\theta}(t)x(t) - u_i(t)), \quad \tilde{x}(0) = 0, \tag{7}$$

where $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$. It should be noted that one cannot reach the bound $\|\tilde{x}\|_{\mathscr{L}_\infty} \leq \sqrt{\theta_{max}/(\lambda_{min}(V)\Gamma)}$ of the nonaugmented system, where $\theta_{max} \triangleq 4\max_{\theta^T\in\Theta}\|\theta\|_2^2$ and $\lambda_{min}(V)$ is the minimum eigenvalue of V, for (7) using the same Lyapunov analysis [12] since it assumed that the dynamics are not directly driven by an input. Additionally, one cannot show the asymptotic result $\lim_{t\to\infty}\tilde{x}(t) = 0$ by application of Barbalat's lemma as $u_i(t)$ is arbitrary and does not depend on $x(t)$, $\tilde{x}(t)$ or $\tilde{\theta}(t)$. Hence, it follows that (6) is not an immediate result for the feedforward augmented system when $u_i(t)$ is excluded from the predictor, since the bounds and the limits rely on these two results. Further details of the stability analysis and derivation of (6) for the nonaugmented system can be found in [12].

III. Iterative Learning Control

ILC architectures, in terms of their relation to existing control loops, can be broadly classified in two groups as parallel and series. In essence, equivalence can be found between the architectures by rearranging input signal nomenclature. The parallel architecture, which we use in our controller, divides the input signal into feedback and feedforward components. In this approach, the learning controller outputs the feedforward signal for the next iteration by processing the error and the feedforward input at the current iteration.

ILC design methods are numerous and include frequency domain, plant inversion, $\mathscr{H}_\infty$ and norm optimization techniques. Frequency methods, whilst only approximating the system (due to finite trial duration), offer simplicity, flexibility and tunability as in classical control. The learning controllers that we use in this paper are thus designed using frequency domain methods.

A. ILC Update Law

A common first order frequency domain ILC algorithm, which we will employ in our controller, is the Q filter and learning function approach:

$$U_{i+1}(s) = Q(s)(U_i(s) + L(s)E_i(s)). \tag{8}$$

In (8), $U_i(s)$ is the ILC input; $Q(s)$ is the Q filter; $L(s)$ is the learning function; $E_i(s)$ is the reference tracking error; and i is the iteration index. In this algorithm, $L(s)$ is designed to maximize learning, while $Q(s)$ is used to limit the bandwidth to robustify the system and for other practical purposes.

B. Stability and Robustness

To simplify the problem from an ILC perspective and free it of nonlinearities, we will design the ILC update law for the reference model (2). Nevertheless, due to the fact that the $\mathscr{L}_1$ controller aims to compensate for the system uncertainty within the bandwidth of $D(s)$, system uncertainty will still exist. Dropping the subscript ref, the closed loop reference system can be described in the s domain as

$$X_i(s) = H(s)U_i(s) + H(s)D(s)K_gR(s) + G(s)\theta X_i(s) + X_{in}(s),$$

where $X_{in}(s) \triangleq (s\mathbb{I} - A)^{-1}x_0$.

Assuming zero initial conditions, the reference model dynamics for ILC are defined as

$$Y_i(s) = P'(s)U_i(s) + P'(s)D(s)K_gR(s), \tag{9}$$

where $P'(s) \triangleq C(\mathbb{I} - G(s)\theta)^{-1}H(s)$.

The $\mathscr{L}_1$ stability condition makes sure that (9) is stable and $(\mathbb{I} - G(s)\theta)^{-1}$ exists. Making use of the matrix identity $(\mathbb{I}+AB)^{-1} = \mathbb{I} - A(\mathbb{I}+BA)^{-1}B$ [15] we rewrite the plant as $P'(s) = P(s)W(s)$, where $P(s) \triangleq CH(s)$; $W(s) \triangleq 1 + \alpha\Delta(s) = \frac{1}{1-\theta G(s)}$; and $\Delta(s) \triangleq \frac{1}{\alpha}\frac{\theta G(s)}{1-\theta G(s)}$ with $\alpha \in \mathbb{R}$ such that $\|\Delta(s)\|_\infty < 1$.

For the ILC update law (8), a sufficient condition for monotonic robust stability, with γ being the convergence rate, is

$$\gamma = \max_{\theta^T \in \Theta} \|Q(s)(1 - L(s)P(s)W(s))\|_\infty < 1. \tag{10}$$

The above condition ensures iteration domain stability of the system by forcing $Q(s)(1 - L(s)P(s)W(s))$ to be a contraction mapping and ensures $\|E_\infty(s) - E_{j+1}(s)\|_\infty \leq \gamma\|E_\infty(s) - E_j(s)\|_\infty$. For causal $Q(s)$ and $L(s)$, this also implies monotonic convergence under the $\mathscr{L}_2$ norm for any finite duration system with rate γ; i.e. $\|e_\infty - e_{j+1}\|_{\mathscr{L}_2} < \gamma\|e_\infty - e_j\|_{\mathscr{L}_2}$.

Observe that θ, and naturally the phase of $1 - L(s)P(s)W(s)$ are unknown; which makes (10) of no practical value. A feasible condition that satisfies (10), however,can be stated as

$$\alpha \leq \frac{\gamma - |Q(j\omega)||1 - L(j\omega)P(j\omega)|}{|Q(j\omega)||L(j\omega)||P(j\omega)|}. \tag{11}$$

To find a tight lower bound on α and invoke (11), we rely on the $\mathscr{H}_\infty$ norm of $G(s)$:

$$|\theta G(j\omega)| \leq \|\theta^T\|_2 \|G(j\omega)\|_2 \leq \varepsilon\|G(s)\|_\infty,$$

where $\varepsilon \triangleq \max_{\theta\in\Theta}\|\theta^T\|_2 = m\sqrt{n}$. Note that $\varepsilon\|G(s)\|_\infty \leq \lambda < 1$. To see this, define the input-output pair $u(t) \in \mathscr{L}_{2e}$ and $x(t) = \begin{bmatrix} x_1(t) & x_2(t) & \dots & x_n(t)\end{bmatrix}^T \in \mathscr{L}_{2e}^n$ for the system $G(s)$. By definition, $\|g_i\|_{\mathscr{L}_1} \leq \|G(s)\|_{\mathscr{L}_1}$, where g_i is the i^{th} element of the impulse response of $G(s)$. Hence,

$$\|(x_i)_\tau\|_{\mathscr{L}_2} \leq \|g_i\|_{\mathscr{L}_1}\|u_\tau\|_{\mathscr{L}_2} \leq \|G(s)\|_{\mathscr{L}_1}\|u_\tau\|_{\mathscr{L}_2}.$$

A proof of the first inequality can be found in [16]. Thus,

$$\|x_\tau\|_{\mathscr{L}_2} = \left[\sum_{i=1}^{n}\|(x_i)_\tau\|^2_{\mathscr{L}_2}\right]^{1/2} \leq \sqrt{n}\|G(s)\|_{\mathscr{L}_1}\|u_\tau\|_{\mathscr{L}_2}. \tag{12}$$

Since $\|G(s)\|_\infty$ is the $\mathscr{L}_2$ norm of the system, (12) implies

$$\|G(s)\|_\infty \leq \sqrt{n}\|G(s)\|_{\mathscr{L}_1}. \tag{13}$$

It follows from (13) and (1) that $\varepsilon\|G(s)\|_\infty \leq \lambda < 1$. Consequently $\alpha > \frac{\varepsilon\|G(s)\|_\infty}{1-\varepsilon\|G(s)\|_\infty}$.

Assuming a stable update law, the iteration domain equilibrium can be expressed as

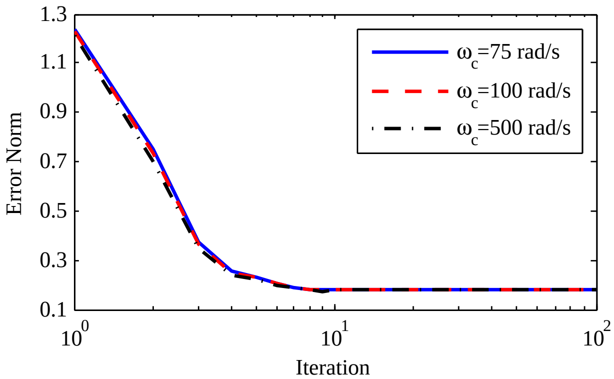
$$U_\infty(s) = \frac{Q(s)L(s)}{1-Q(s)(1-L(s)P(s)W(s))}(1 - F(s))R(s),$$
$$E_\infty(s) = \frac{1-Q(s)}{1-Q(s)(1-L(s)P(s)W(s))}(1 - F(s))R(s),$$

where $F(s) \triangleq P'(s)D(s)K_g$. The complete $\mathscr{L}_1$-ILC scheme can be seen in fig. 1. We refer the readers to [1] for further details on frequency domain ILC design methods.

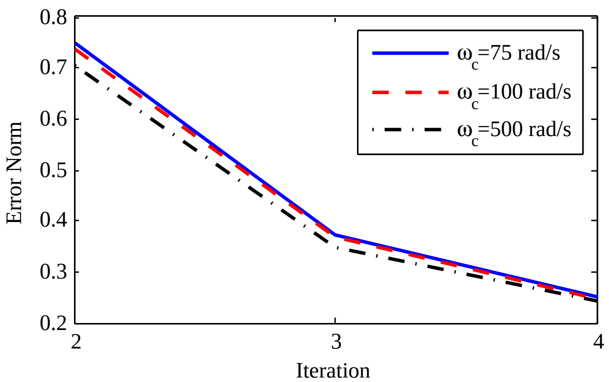
IV. Design Trade-offs

To examine the trade-offs between iteration and time domain properties we will make use of the following inequalities:

$$|W(j\omega)| \geq \frac{1}{|1 - \theta H(j\omega)| + |D(j\omega)||\theta H(j\omega)|}.$$



(a) Iteration series in 100 trials



(b) Close-up on iterations 2-4

Fig. 2: Effect of feedback bandwidth on ILC performance with $K = 0.80$

$$\frac{\gamma}{|Q(j\omega)|} \geq |1 - |L(j\omega)P(j\omega)W(j\omega)||.$$

It follows directly that

$$|L(j\omega)||P(j\omega)| \leq \left(\frac{\gamma}{|Q(j\omega)|} + 1\right) \times (|1 - \theta H(j\omega)| + |D(j\omega)||\theta H(j\omega)|).$$

Both $D(s)$ and $Q(s)$ describe the performance-robustness trade-offs in their respective domains. Thus, generally speaking, we can deduce the following design trade-offs:

1) Increasing the bandwidth of $D(s)$ decreases the minimum γ that satisfies (11), i.e. better iteration domain transients. Indirectly, a higher bandwidth also results in better iteration domain robustness, thereby leaving the possibility of higher gain Q filters for enhanced performance: As the $\mathscr{L}_1$ filter bandwidth increases, the minimum γ in (11) becomes bounded further away from 1 and naturally, α decreases since $G(s) \triangleq H(s)(1 - D(s))$. As a result, the designer can tune $Q(s)$ to increase its bandwidth and minimize the converged error.
2) Decreasing the bandwidth of $Q(s)$ decreases the minimum allowable γ that would satisfy (11), which signifies increased iteration domain robustness. This further implies that one can use a lower gain $D(s)$ for a feedback system with better stability margins: Because $Q(s)$ has a lower gain, there exists a higher value of α satisfying (11) for the original value of γ.

It thus makes sense to summarize the design trade-offs for the combined adaptive-learning controller as that of performance versus robustness. Intuitively, this is to be expected as increasing the passband of $D(s)$ decreases parametric uncertainty as $W(s) = [1 - \theta H(s)(1 - D(s))]^{-1}$, which is the desired result from an ILC perspective.

V. Simulation Results

To illustrate the effects of $D(s)$ and $Q(s)$ on system performance and gain further insight into the controller, we consider the plant

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

and let $\Theta = \left\{ v \in \mathbb{R}^2 : \|v\|_\infty \leq 5 \right\}$ [12]. The ILC update law is defined in the frequency domain as (8) with $Q(s) = K\frac{50}{s+50}$ and $L(s) = 2.8s$. The DC gain of $Q(s)$ is used to robustify the update law for plant uncertainties due to the first order low-pass filter $D(s)$ and the uncertain parameter θ. For the sake of demonstration, $\theta = \begin{bmatrix} 4 & -4.5 \end{bmatrix}$ in fig. 2 and fig. 3. Note that the open loop plant is unstable when θ is chosen as such. In the implementation of the adaptation law, we choose $\Gamma = 1 \times 10^6$. For the state predictor, we let $K_{sp} = 0$.

Fig. 2 shows the effect of $D(s)$ on iteration domain performance where the performance metric is the $\mathscr{L}_2$ norm of the error. Here, the nonunity DC gain of 0.80 ensures iteration domain stability for $\mathscr{L}_1$ cutoff frequencies (ω_c) higher than 75 rad/s. For the cutoff frequencies 75, 100 and 500; the lower bound on α is 0.117, 0.085 and 0.016, respectively. Notice that as opposed to our findings in [11], a higher bandwidth implies better performance; even though the difference is minimal in this case. The effect of the bandwidths of both filters can be seen in fig. 3: Lower converged errors can be achieved by increasing the bandwidth of both filters, albeit at the cost of slower transient behavior. Nonetheless, performance enhancement comes with decreased stability

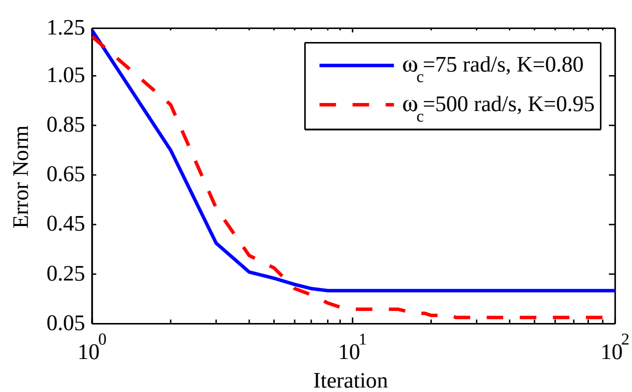
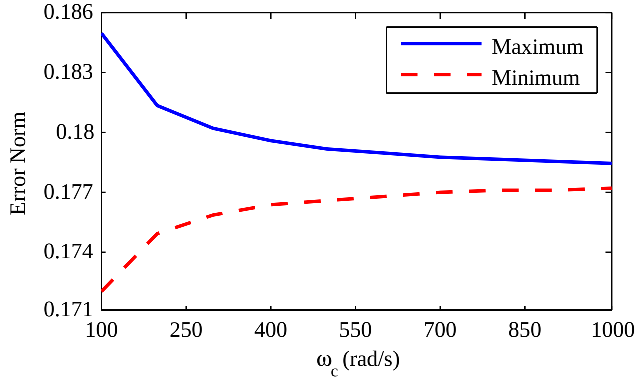
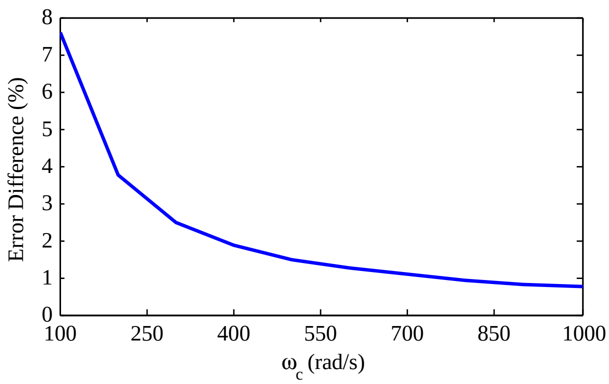


Fig. 3: Effect of Q and $\mathscr{L}_1$ filter bandwidths on learning performance



(a) Maximum and minimum converged errors in the uncertainty set



(b) Maximum difference in converged errors in the uncertainty set

Fig. 4: Marginal performance metrics for varying feedback bandwidths with $K = 0.80$

margins; in a given system (A, B, C) with parameter θ time delays and/or other unmodeled dynamics will always exist.

An important point of interest is the monotonicity of the controller against varying values of θ. Due to the complicated dynamics involved in the equilibrium signals $U_\infty(s)$ and $E_\infty(s)$, it is difficult to come up with an analytical form that tightly bounds the transients due to a change in the parameter vector θ^T, or that describes the difference in converged error values. These have been evaluated numerically in fig. 4. Maximum and minimum values of the converged error in the uncertainty set Θ have been plotted versus ω_c. Note that both curves converge asymptotically towards the same value as the bandwidth of $D(s)$ increases. The maximum difference in converged errors, i.e. $((\max_{\theta^T \in \Theta} \|e_\infty\|_{\mathscr{L}_2} / \min_{\theta^T \in \Theta} \|e_\infty\|_{\mathscr{L}_2}) - 1)$, is less than 7.6% for $\omega_c \geq 100$rad/s. As can be seen, increasing the cutoff frequency of the filter even from 100 to 200 rad/s dramatically increases the monotonicity of the controller, based on the assumed difference between converged errors in response to marginal values of θ. Iteration series of the system for the parameters $\theta_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$, $\theta_1 \triangleq [\arg\min_{\theta^T \in \Theta} \|e\|_{\mathscr{L}_2}]^T = \begin{bmatrix} 5 & 5 \end{bmatrix}$ and $\theta_2 \triangleq [\arg\max_{\theta^T \in \Theta} \|e\|_{\mathscr{L}_2}]^T = \begin{bmatrix} -5 & -5 \end{bmatrix}$ are shown in fig. 5. Despite the relatively low bandwidth of $D(s)$ ($\omega_c = 100$ rad/s), the iteration dynamics for all parameters are highly similar. The $\mathscr{L}_2$ error norms for θ_1 (0.117) and θ_2 (0.186), which correspond to a 7.5% difference, are equal to the values in fig. 4, albeit the infinite horizon assumption of s domain analysis.

An equally significant property of the controller is its response to abrupt changes in the uncertain parameter. The superiority of the nonaugmented $\mathscr{L}_1$ feedback based ILC over another ILC architecture with linear quadratic regulation (LQR) and integral control was previously shown in [11]. Transients of the system when the uncertain parameter is changed from θ_1 to θ_2, as quantified by the percentage deviation from the converged error when $\theta = \theta_1$, is shown in fig. 6. While the approximately 42.0% transient for $\omega_c = 100$rad/s may seem large at first glance, it is worth noting that system transients were shown to be much larger in a comparable case for the LQR-Integral controller. Furthermore, similar to the trend in fig. 4, additional reduction in transients can be achieved by increasing the bandwidth of the $\mathscr{L}_1$ filter.

Finally, fig. 7 presents a scenario where the initial parameter θ_0 is changed to θ_1 at the 11th iteration and to θ_2 at the 51st iteration. In spite of the noticeable transients at these points, the system remains fairly stable and goes back to equilibrium in a couple of iterations. A transient of 42.4% at the 51st iteration verifies the claims in fig. 6, while the converged error metrics for θ_1 and θ_2 are the same as those in fig. 4 and fig. 5.

VI. Conclusions

In this paper, we presented a rigorous stability analysis and performance evaluation of the iterative learning controller with the modified $\mathscr{L}_1$ adaptive feedback controller. A simple robust stability condition that generalizes our previous stability analysis to $\mathscr{L}_1$ filters of arbitrary bandwidth was given. Performance trade-offs between time and iteration domains were examined and numerically evaluated. Our findings indicate that the augmentation of the $\mathscr{L}_1$ controller with the feedforward input results in an overall controller that focuses on minimizing the reference tracking error. The design trade-off thus simplifies to the performance-robustness trade-off that can be seen in the respective domains of both controllers. Monotonicity of the controller makes it a

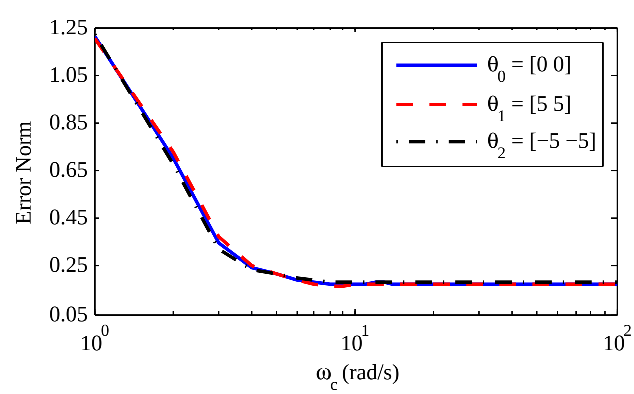


Fig. 5: Iteration series for varying values of the uncertain parameter with $K = 0.80$ and $\omega_c = 100$ rad/s

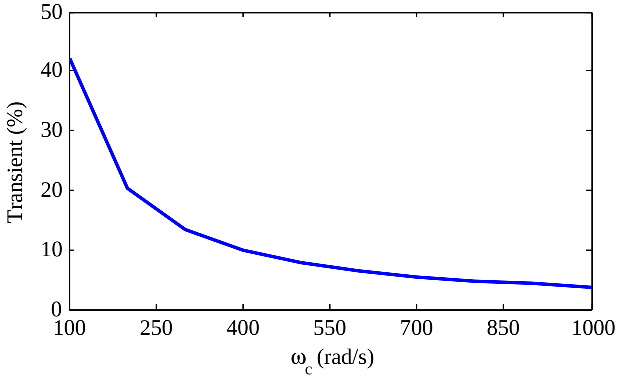


Fig. 6: Effect of the feedback filter bandwidth on system transients with $K = 0.80$

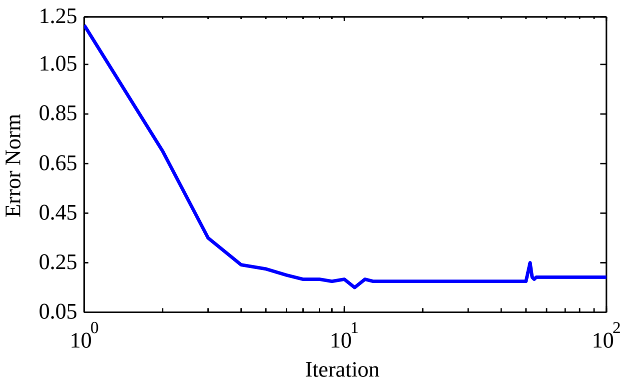


Fig. 7: Transients due to abrupt changes in the uncertain parameter with $K = 0.80$ and $\omega_c = 100$ rad/s

promising performance enhancement scheme for applications with large parametric uncertainties.

Future work includes extending the results to the output feedback case and multiple-input multiple-output (MIMO) LTI systems, along with experimental verification of the proposed architecture.

References

[1] D. Bristow, M. Tharayil, and A. Alleyne, "A survey of iterative learning control," *Control Systems, IEEE*, vol. 26, no. 3, pp. 96–114, 2006.

[2] K. Moore, *Iterative Learning Control for Deterministic Systems*. London: Springer-Verlag, 1993.

[3] H.-S. Ahn, Y. Chen, and K. Moore, "Iterative learning control: Brief survey and categorization," *Systems, Man, and Cybernetics, Part C: Applications and Reviews, IEEE Transactions on*, vol. 37, no. 6, pp. 1099–1121, 2007.

[4] D. de Roover and O. H. Bosgra, "Synthesis of robust multivariable iterative learning controllers with application to a wafer stage motion system," *International Journal of Control*, vol. 73, no. 10, pp. 968–979, 2000.

[5] J. van de Wijdeven, T. Donkers, and O. Bosgra, "Iterative learning control for uncertain systems: Robust monotonic convergence analysis," *Automatica*, vol. 45, no. 10, pp. 2383–2391, 2009.

[6] J.-H. Moon, T.-Y. Doh, and M. J. Chung, "A robust approach to iterative learning control design for uncertain systems," *Automatica*, vol. 34, no. 8, pp. 1001–1004, 1998.

[7] M. French and E. Rogers, "Nonlinear iterative learning by an adaptive lyapunov technique," in *Decision and Control, 1998. Proceedings of the 37th IEEE Conference on*, vol. 1, 1998, pp. 175–180.

[8] W. Seo, B. Park, and J. Lee, "Intelligent learning control for a class of nonlinear dynamic systems," *Control Theory and Applications, IEE Proceedings*, vol. 146, no. 2, pp. 165–170, 1999.

[9] J. Y. Choi and J. Farrell, "Observer-based backstepping control using online approximation," in *American Control Conference, 2000. Proceedings of the 2000*, vol. 5, 2000, pp. 3646 –3650.

[10] H.-S. Ahn, K. L. Moore, and Y. Chen, "Stability analysis of discrete-time iterative learning control systems with interval uncertainty," *Automatica*, vol. 43, no. 5, pp. 892–902, 2007.

[11] K. Barton, S. Mishra, and E. Xargay, "Robust iterative learning control: $\mathscr{L}_1$ adaptive feedback control in an ILC framework," in *American Control Conference*, 2011, pp. 3663–3668.

[12] C. Cao and N. Hovakimyan, *$\mathscr{L}_1$ Adaptive Control Theory: Guaranteed Robustness with Fast Adaptation*. Philedelphia, PA: Society for Industrial and Applied Mathematics, 2010.

[13] ——, "Design and analysis of a novel $\mathscr{L}_1$ adaptive controller, part I: Control signal and asymptotic stability," in *American Control Conference*, 2006, pp. 3397–3402.

[14] J.-B. Pomet and L. Praly, "Adaptive nonlinear regulation: estimation from the lyapunov equation," *Automatic Control, IEEE Transactions on*, vol. 37, no. 6, pp. 729–740, 1992.

[15] S. Searle, *Matrix Algebra Useful for Statistics*. New Jersey: John Wiley and Sons, 2002.

[16] H. Khalil, *Nonlinear Systems*. Eaglewood Cliffs, NJ: Prentice-Hall, 2002.